

### THE LEVEL PAYMENT PROBLEM

1. Consider the recurrence

$$x_n = \begin{cases} 1.1x_{n-1} - 1000 & \text{if } n \geq 1 \\ 12000 & \text{if } n = 0. \end{cases}$$

Describe a situation this recurrence models. Use your axe to compute  $x_0$  up to  $x_{50}$ . Graph it and sketch the graph.

2. What happens to the recurrence if we let  $x_0 = 8000$ ?

3. If we allow  $x_0$  to vary, what kinds of behaviour does the recurrence exhibit. Classify these behaviours with a simple rule. Show sample graphs. Can you choose  $x_0$  so that the sequence remains constant? If so, explain, in common-sense terms, why this happens.

4. Let us suppose that  $x_0$  is given, that  $a$  and  $b$  are constants, and that  $x_n = ax_{n-1} + b$ . Write out the terms  $x_1$  through  $x_5$ . Clean them up as best you can. Detect a pattern and write a formula for  $x_n$ .

5. Write a recurrence for the following situation. A loan has an initial principal of  $P_0$  and an interest rate of  $r$  per time period. An periodic payment of  $Q$  dollars is made starting with the first time period. Use the result of the last problem to write a formula for computing  $x_n$ .