

Midterm Test
discreteG, 0405, T1

Mandatory Problems, Part I, You have no choice!

1. Determine the number of nonnegative integer solutions (x, y, z) to $x + y + z = 15$. Determine the number of these solutions satisfying $x \geq 3$, $y \geq 4$ and $z \geq 1$.

2. *Comment on the validity of the solution to the following problem.* A committee of 10 people is to be chosen from 8 men and 6 women so that the committee has at least 4 women. Cooley says it can be done $\binom{6}{4} \binom{10}{6}$ ways. She argues that you must first select 4 women from the 6 and you can then select 6 more members freely from the remaining people to fill out the committee. She claims the counting will include all committees of 4, 5, or 6 women and none with fewer.

3. Use mathematical induction to prove that the sum of the first n odd positive integers is n^2 . Produce a formula with appropriate Σ -notation.

4. Two students are asked to determine the number of ways of arranging 6 Qs, 7 Rs, 3 Ss and 3 Ts in a "word" of length 19. Blevins completes this assignment and gets the result

$$\frac{19!}{3!3!6!7!}$$

and Wilson does the same thing and he gets

$$\binom{19}{3} \binom{16}{3} \binom{13}{7} \binom{6}{6}.$$

Explain how Blevins and Wilson arrived at their (seemingly) disparate solutions.

5. How many strings of length 10 can be made from the word MISSISSIPPI?

Part II, You have a choice! Pick 5!

6. A valet has keys for 10 cars and 10 owners waiting for a car. In how many ways can he distribute the keys so that exactly 4 owners get their keys back and so the rest get wrong keys?

7. A basketball tournament has 16 teams. How many possible ways are there to pair the teams off for the first round?

8. Show that if ten nonnegative integers has sum 101, there must be a trio of integers with the ten that has a sum of at least 31.

9. Find the number of ways to make change of a \$50 bill with twenties, tens, fives and ones.

10. Find a formula for $\sum_{k=1}^n \frac{1}{k(k+1)}$. Use induction to verify what you have found.

11. A poll is taken of 600 S&Mers about three favourite restaurants: Cosmic, Elmo's and Randy's Pizza. Three students disliked all of these restaurants and 500 liked all three. 520 students liked Elmo's and Randy's, and the same number liked Cosmic and Randy's. Ten students only liked Cosmic, 12 only liked Elmo's and 5 only liked Randy's. How many students liked Cosmic and Elmo's?

12. A staircase has n stairs; denote by s_n the number of ways to ascend the stairway if you can go up 1, 2 or 3 steps at a time. Write a recursive formula for s_n . Can you write an explicit formula? Compute s_n for $1 \leq n \leq 10$.